

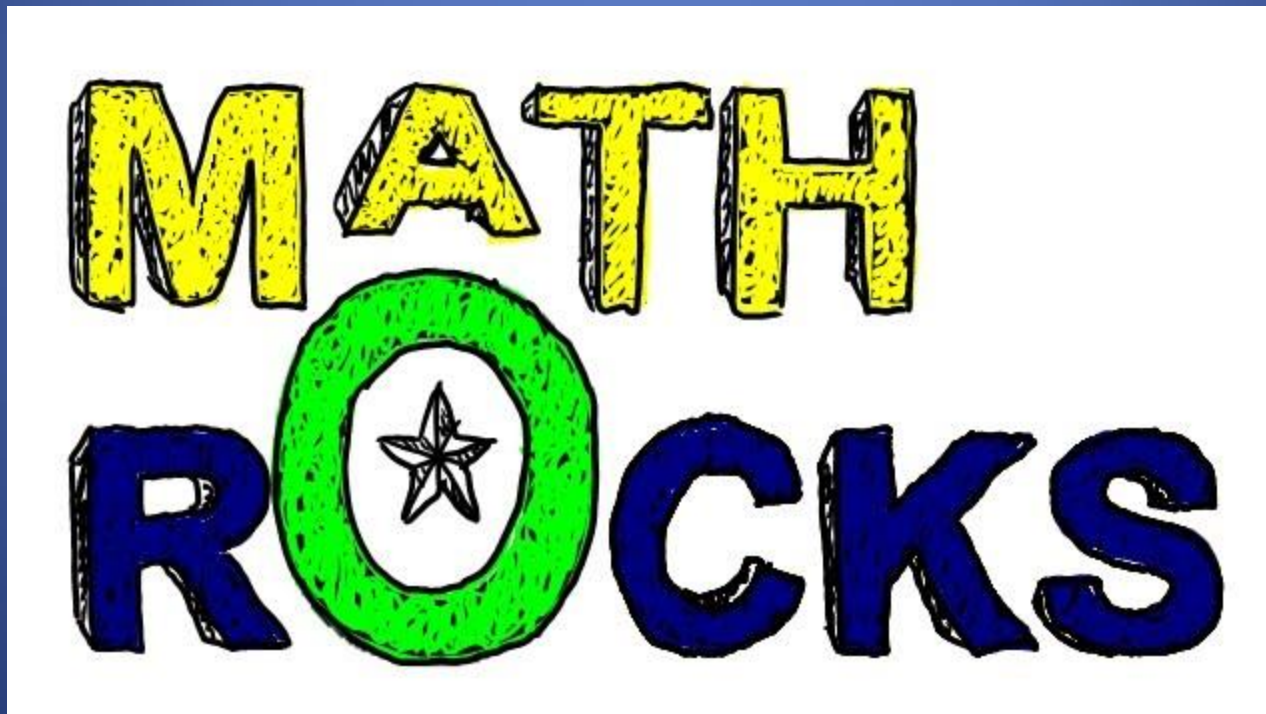
# **SPRING 2015 SRBI SYMPOSIUM II**

## **Instructional Practices in Mathematics**

**Cay Freeman  
Math Intervention Teacher  
Sage Park Middle School, Windsor**

First, you must understand:

I think that...



# Goals for this session:

Participants will....

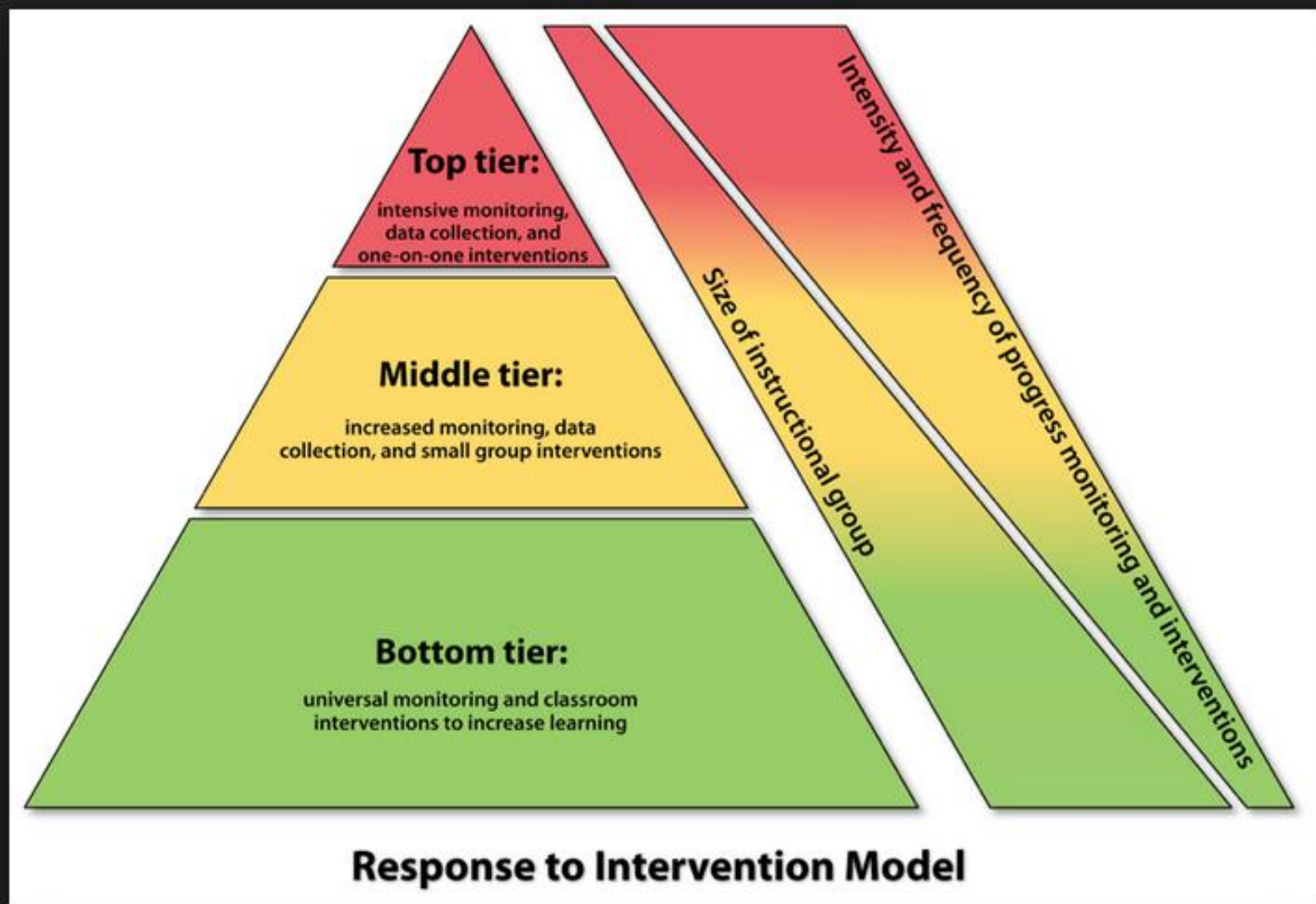
- Review RTI model
- Experience what it's like to be a student struggling with math
- Learn the components of an SRBI Math program
- Review recommendations from the What Works Clearinghouse
- Learn about resources to aid in your SRBI efforts

# Goals for this session:

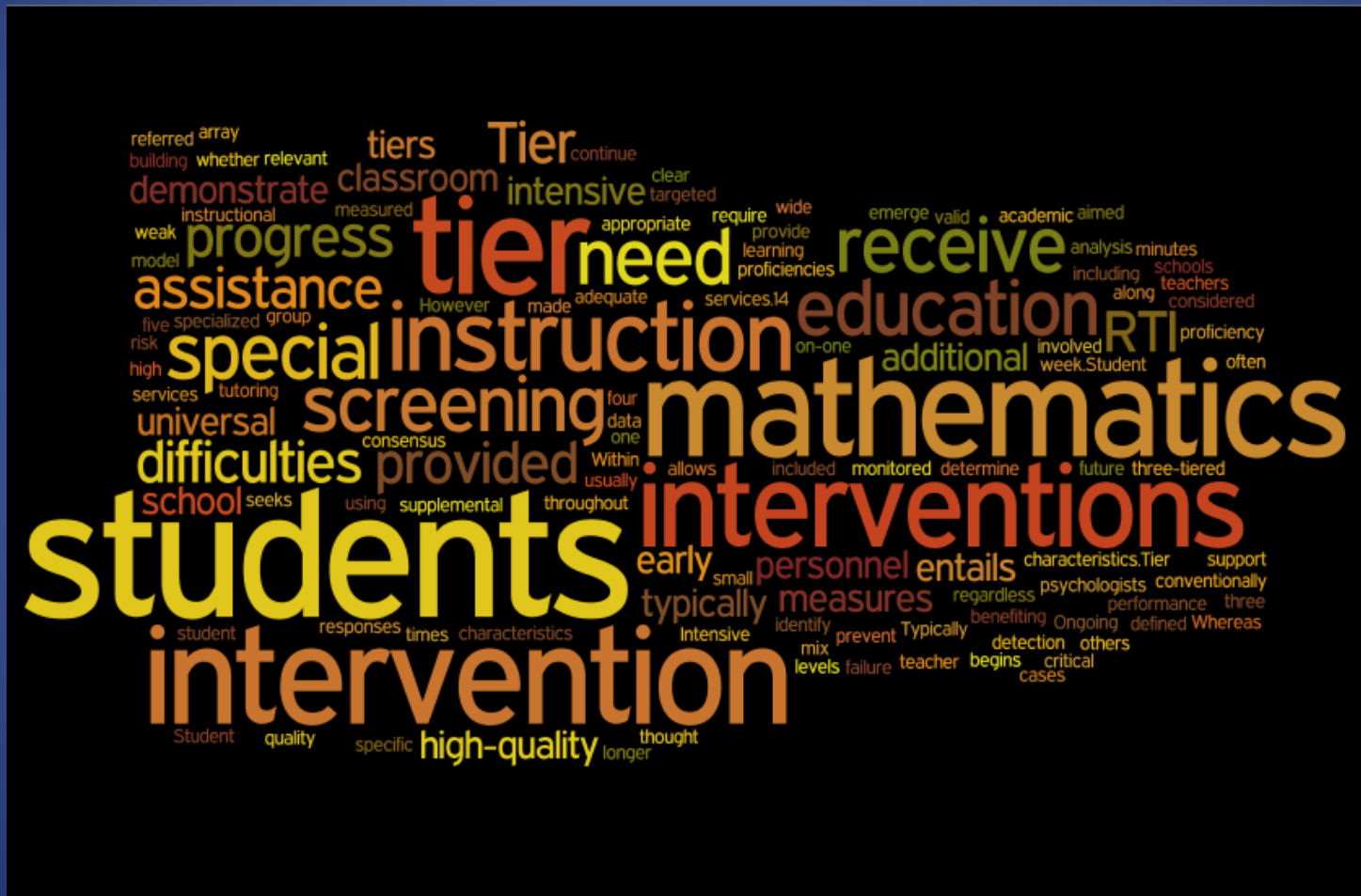
Participants will....

- Learn critical factors to consider when planning and making ongoing adjustments to teaching to ensure student achievement
- Learn promising instructional practices targeting Tier 2 and Tier 3 math students, with supporting research basis

# Basic Review of SBRI Model:



# SRBI planning and decision making?



# Think of that student who:

- thinks she's a dummy because she just doesn't get it when everyone else seems to
- acts out because it's easier than showing he doesn't understand
- copies everything down and looks like she knows what she's doing, but cannot tell you what it means
- is frustrated because he thought he knew the rules, but the rules keep changing (in math)

# Let's walk a mile in their shoes...

- $23 + 12 = ?$

	Hundreds ( $10^2$ )	Tens ( $10^1$ )	Ones ( $10^0$ )
23		2	3
<u>+12</u>		1	2
35		3	5

- $23 + 12 \neq 35$ ? What? Why not?????

- $23 + 12 = 101$  What???

The magic of **Base 4**:

	Sixteens ( $4^2$ )	Fours ( $4^1$ )	Ones ( $4^0$ )
23		2	3
<u>+12</u>		1	2
101		3	<del>5</del>

4 ones

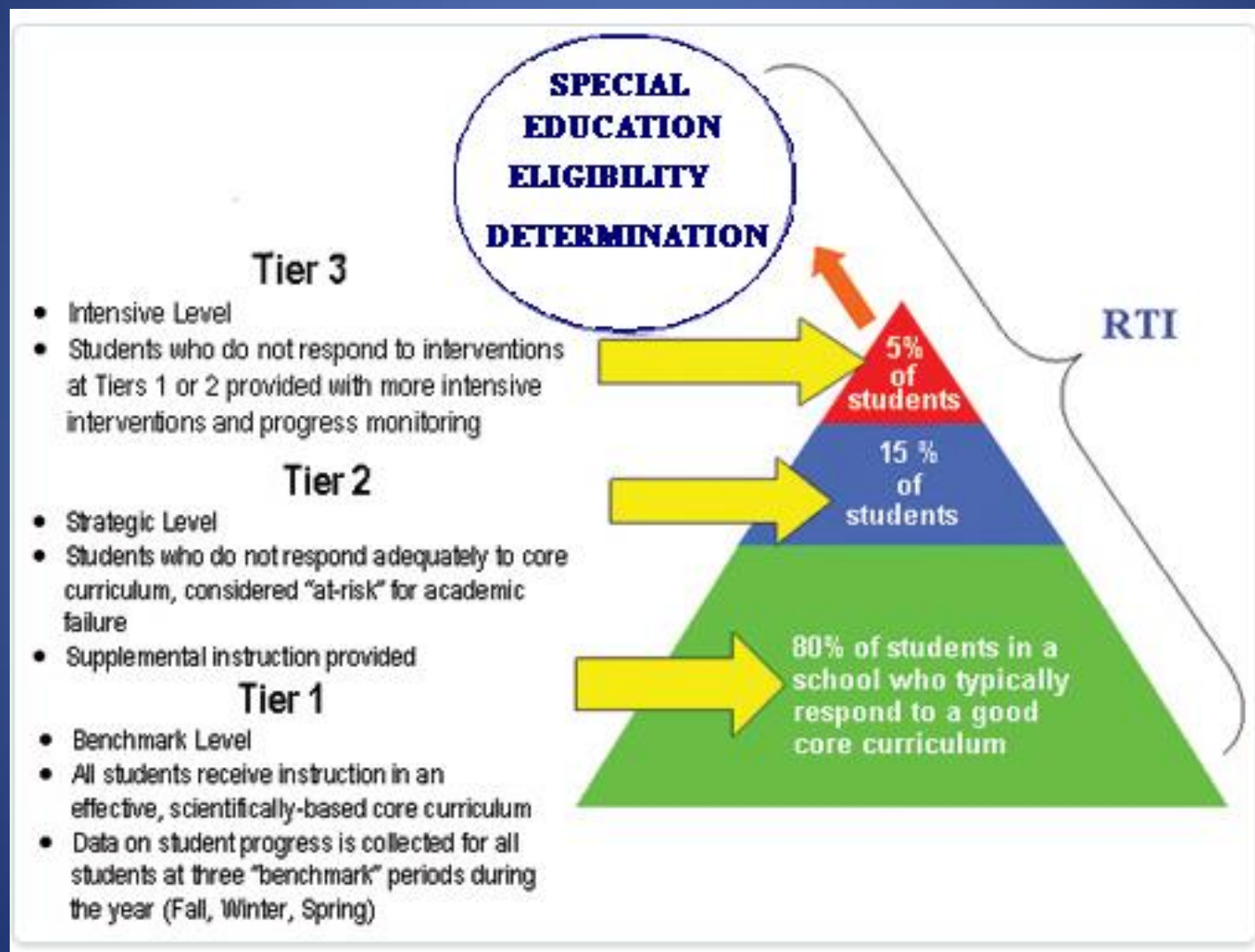
		<del>4</del>	1
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4 fours

	1	0	1
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# The RTI Model: 3 Tiers



# Tier 1: Benchmark Level

- Who: all students
  - Will be successful for approx. 80% of students
- What: high quality, empirically supported curriculum and instruction
- When: regularly scheduled classroom instruction
- How assessed: progress is monitored at minimum three times a year
  - Benchmark screening during the Fall, Winter and Spring

# Tier 2: Supplemental Level

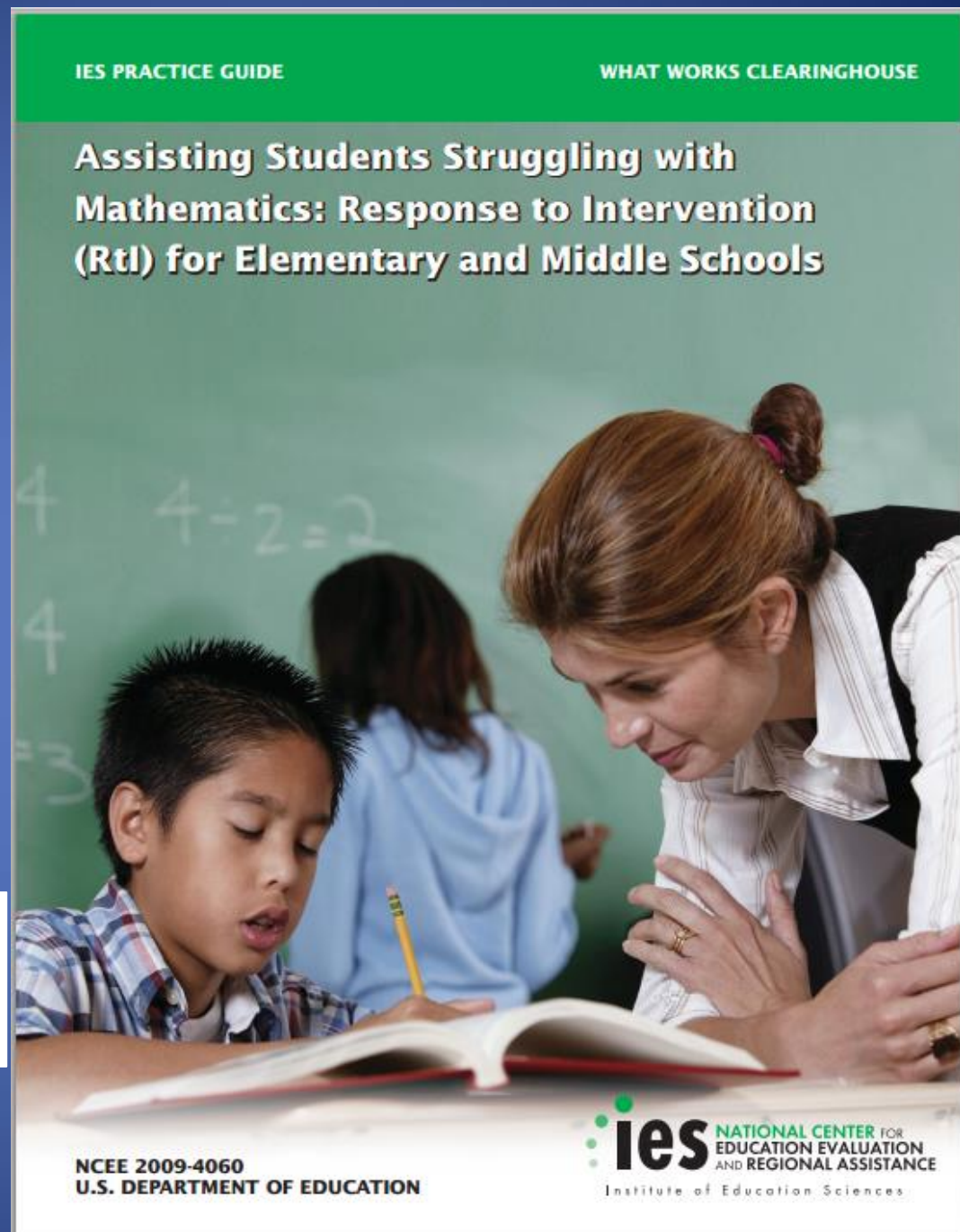
- Who: students not making adequate progress in the Tier 1 core curriculum –
  - 10% - 15% of students
  - **Small group** instruction
- What: research-based instruction and strategies that are **matched to student need** and **focused on core competencies** that support, enhance, or supplement Tier 1 instruction
- When: in addition to core instruction, **several times a week**
- How assessed: on-going (**1-2 times a month**) progress monitoring to measure student response to intervention and to guide decision making

# Tier 3: Intensive Level

- Who: students not making adequate progress in the Tier 1 core and Tier 2 curriculum –
  - Approx. 5% of students
  - **Individual** and small group instruction
- What: research-based **intense intervention** and strategies that are matched to **individual student need and progress** and focused on core competencies
- When: in addition to core instruction, **every day**
- How assessed: on-going (**1-2 times per week**) progress monitoring to measure student response to intervention and to guide decision making

My “Go-to”  
Resource for  
Math SRBI:

[http://ies.ed.gov/ncee/wWc/pdf/practice\\_guides/rti\\_math\\_pg\\_042109.pdf](http://ies.ed.gov/ncee/wWc/pdf/practice_guides/rti_math_pg_042109.pdf)



Each recommendation includes:

1. Brief Summary
2. How to carry out the recommendation
3. Potential roadblocks and solutions

**Table 2. Recommendations and corresponding levels of evidence**

Recommendation	Level of evidence
Tier 1	
1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.	Moderate
Tiers 2 and 3	
2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.	Low
3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	Strong
4. Interventions should include instruction on solving word problems that is based on common underlying structures.	Strong
5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.	Moderate
6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate
7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.	Low
8. Include motivational strategies in tier 2 and tier 3 interventions.	Low

Source: Authors' compilation based on analysis described in text.

# Recommendation 2:

- Instructional materials should focus intensely on in-depth treatment of:

## Grades K-3:

- Number sense
- Place value of whole numbers
- Operations with whole numbers

## Grades 4-8:

Use an understanding of whole numbers to build a conceptual framework and work with rational numbers, which are critical for future success in math

Cover fewer topics in more depth and with coherence

Intervention curriculum should not be over-simplified

# Focus on High-Impact Work:

## HIGHLIGHTS OF MAJOR WORK IN GRADES K–8

K–2	Addition and subtraction – concepts, skills, and problem solving; place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions



# Common Core State Standards

Table 1. Progress to Algebra in Grades K–8

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction		Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions		
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Represent and solve problems involving addition and subtraction	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Compare numbers		Add and subtract within 20	Multiply & divide within 100	Use place value understanding and properties of operations to perform multi-digit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Analyze proportional relationship and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Add and subtract within 20	Understand place value	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Work with numbers 11–19 to gain foundations for place value	Work with addition and subtraction equations	Use place value understanding and properties of operations to add and subtract	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
	Extend the counting sequence	Measure and estimate lengths in standard units	Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects			Represent and analyze quantitative relationships between dependent and independent variables		Use functions to model relationships between quantities
	Understand place value	Relate addition and subtraction to length	Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*			
	Use place value understanding and properties of operations to add and subtract							
	Measure lengths indirectly and by iterating length units							

# Should my SRBI program be aligned to the core curriculum?

- Alignment with the core curriculum is not as critical as ensuring that instruction **builds students' foundational proficiencies.**
- Tier 2 and tier 3 instruction often focuses on prerequisite skills that **allow the student to access the general curriculum**, and are determined by the students' rate of progress.

# SRBI

## Planning Tool:

Last year:

Focus was on

building strong

foundational

skills to support

Tier 1 content

Math SRBI Planning Sheet for 2013-2014			
		Tier 2	Tier 3
1 <sup>st</sup> Quarter	6 <sup>th</sup>	Fraction Concepts: Do the Math Now! Fraction Fundamentals	Whole Number Computation: Do the Math Now! Multiplication and Division
	7 <sup>th</sup>	Fraction Concepts: Equivalence, order, compare, number line, mixed to improper, +/- like denominators	Fraction Concepts: Do the Math Now! Fraction Fundamentals Unit 6
	8 <sup>th</sup>	Fraction Computation: Foundations for Algebra: Fractions and Decimals	Fraction Concepts: Do the Math Now! Fraction Fundamentals Units 8 and 9
2 <sup>nd</sup> Quarter	6 <sup>th</sup>	Fraction Concepts: Do the Math Now! Fraction Fundamentals	Whole Number Computation: Do the Math Now! Multiplication and Division
	7 <sup>th</sup>	Fraction Computation: Foundations for Algebra: Fractions and Decimals	Fraction Concepts: Do the Math Now! Fraction Fundamentals Unit 7
	8 <sup>th</sup>	Fraction Computation/ Ratios+ Proportions Foundations for Algebra: Fractions and Decimals	Fraction Computation: Do the Math Now! Fraction Fundamentals Foundations for Algebra: Fractions/ Decimals
3 <sup>rd</sup> Quarter	6 <sup>th</sup>	Fraction Computation: Do the Math Now! Fraction Fundamentals	Whole Number Computation: Do the Math Now! Multiplication and Division
	7 <sup>th</sup>	Fraction / Decimal Computation: Foundations for Algebra: Fractions and Decimals	Fraction Concepts: Do the Math Now! Fraction Fundamentals Unit 8
	8 <sup>th</sup>	Integers and Algebra: Foundations for Algebra: Integers, Equations	Fraction Computation: Foundations for Algebra: Fractions/ Decimals
4 <sup>th</sup> Quarter	6 <sup>th</sup>	(Finish) Fraction & (Begin) Decimal Computation: Do the Math Now! Fraction Fundamentals	Fraction Concepts: Do the Math Now! Fraction Fundamentals
	7 <sup>th</sup>	Integers and Algebra: Foundations for Algebra: Integers, Equations	Fraction Concepts: Do the Math Now! Fraction Fundamentals Unit 9
	8 <sup>th</sup>	Integers and Algebra: Foundations for Algebra: Integers, Equations	Ratios/Proportions / Integers Foundations for Algebra: Fractions/ Decimals Foundations for Algebra: Integers, Equations

# SRBI Planning Tool:

	1 <sup>st</sup> Quarter	2 <sup>nd</sup> Quarter	3 <sup>rd</sup> Quarter	4 <sup>th</sup> Quarter
Grade 6, Tier 2	Fraction Concepts →		Fraction Computation → (Begin Decimal Computation, if time)	
Grade 6, Tier 3	Whole Number Computation →			Fraction Concepts
Grade 7, Tier 2	Fraction Concepts	Fraction Computation	Frac/Dec. Computatōn	Ratios/Proportions
Grade 7, Tier 3	Fraction Concepts →			
Grade 8, Tier 2	Fraction Computation	Fr.Comp./Ratios/Prop.	Integer Computation	Integers/ Algebra eq.
Grade 8, Tier 3	Fraction Concepts	Fraction Computation	Fraction Computation	Ratios/ <u>Propor</u> /Integers

SRBI  
Planning Tool:  
This year:  
Focus is on  
remediating  
Tier 1 content  
from the prior  
marking period

Math SRBI Planning Sheet for 2014-2015			
**Tentative**			
	Tier 2 (not meeting gr level standard on unit assessments)	Tier 3 (more than 1 yr below grade level)	
1 <sup>st</sup> Quarter	6 <sup>th</sup>	Prepping for Decimal Computation (reviewing gr 5 content)	ST Math time (gr 6 content), Fastt Math, & remediation curriculum (tbd)
	7 <sup>th</sup>	Prepping for Fraction Computation (reviewing gr 6 content)	ST Math (Secondary Intervention) & Fastt Math
	8 <sup>th</sup>	Prepping for The Number System (reviewing gr 7 content)	ST Math (Secondary Intervention) & Fastt Math
2 <sup>nd</sup> Quarter	6 <sup>th</sup>	Re-teaching standards from Understanding +/- #s, & Add/Sub/Mult Decimals	ST Math time (gr 6 content), Fastt Math, & remediation curriculum (tbd)
	7 <sup>th</sup>	Re-teaching add/sub/mult/div rational #s	ST Math (Secondary Intervention) & Fastt Math
	8 <sup>th</sup>	Re-teaching Pythagorean Theorem & Real Numbers	ST Math (Secondary Intervention) & Fastt Math
3 <sup>rd</sup> Quarter	6 <sup>th</sup>	Re-teaching standards from Division (whole #/dec/frac) & Expression & Eqs	ST Math time (gr 6 content), Fastt Math, & remediation curriculum (tbd)
	7 <sup>th</sup>	Re-teaching Applications of Proportions & Connecting Tables/Graphs/Eqs	ST Math (Secondary Intervention) & Fastt Math
	8 <sup>th</sup>	Re-teaching Volume of Cylinders/Cones/Spheres & Congruence & Similarity	ST Math (Secondary Intervention) & Fastt Math
4 <sup>th</sup> Quarter	6 <sup>th</sup>	Re-teaching standards from Geometry (SA & Volume) & Ratios, Rates, & Percents	ST Math time (gr 6 content), Fastt Math, & remediation curriculum (tbd)
	7 <sup>th</sup>	Re-teaching Expressions & Equations & 2d & 3d Geometry & Measurement	ST Math (Secondary Intervention) & Fastt Math
	8 <sup>th</sup>	Re-teaching Linear Relationships	ST Math (Secondary Intervention) & Fastt Math

# Recommendation 3:

- Instruction should be systematic and explicit :
  - Instruction should gradually build proficiency systematically:
    - introduce concepts in a logical order using logical #s
    - explore multiple methods to arrive at the solution
    - provide students with numerous applications of each concept and skill
    - Demonstrate proficient problem solving with sufficient models

# Explicit Instruction includes:

- Verbalization of the thought processes and the **reasons behind math** procedures, formulas, and problem-solving methods
  - Teachers should be knowledgeable enough to anticipate and **address misconceptions**
  - Include numerous clear models of easy and difficult problems, with accompanying **teacher think-alouds**
    - What are we preparing our students for?

# Explicit Instruction includes:

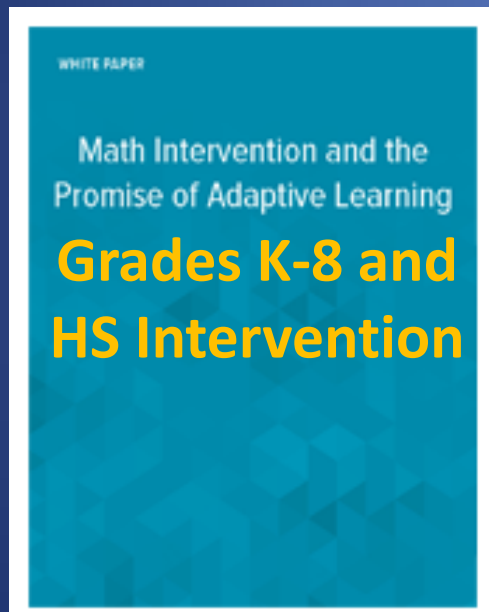
- Guided, scaffolded, **extensive practice**
  - Teacher should ask students to communicate the strategies they are using to complete each step of the process and provide **reasons for their decisions**
    - CCSS Math Practice #3: “Construct viable arguments and critique the reasoning of others.”
- Teachers provide immediate, specific, actionable **corrective feedback** with opportunities to correct errors (with guidance, as needed)
- Frequent, cumulative review



# Individualized, Extensive Practice with Corrective Feedback:



<https://www.tenmarks.com/>



<http://www.dreambox.com/math-intervention>

Designed for the New Standards

Every Standard, Unpacked

Connected to Math Practice Standards

Tied to Content Progression

Cover Each Learning Objective



Constructed Responses and Reasoning Questions

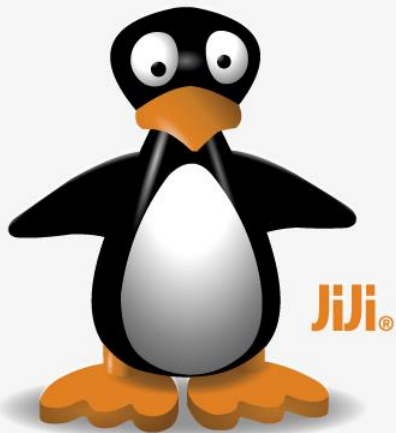
Scaffolded Instruction through Hints and Videos

Assessments with Recommendations

**Grades 1 – Algebra 2 and Geometry**

# New Study Shows Benefits of Visual, Game-Based Math: *EdWeek* 12/08/14

ST Math

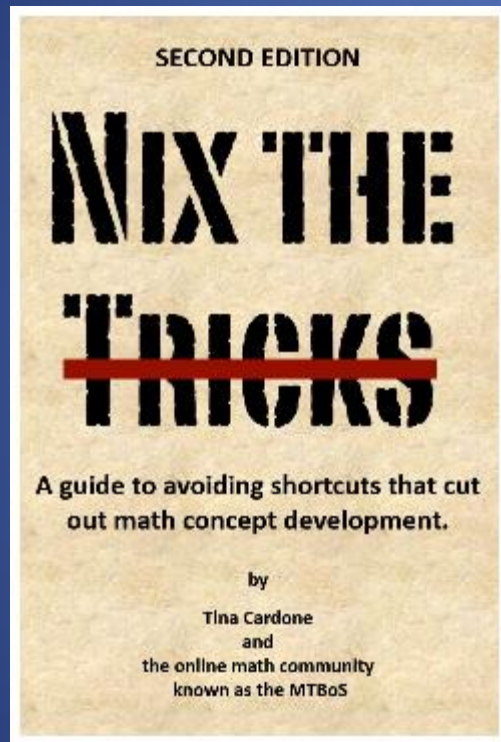


[http://blogs.edweek.org/edweek/on\\_innovation/2014/12/new\\_study\\_shows\\_benefits\\_of\\_visual\\_game-based\\_math.html](http://blogs.edweek.org/edweek/on_innovation/2014/12/new_study_shows_benefits_of_visual_game-based_math.html)

ST Math has six distinct advantages:

1. **Game-based:** engaging and challenging learning games that promote persistence and performance
2. **Instructional:** the games promote hypothesis development and testing; failure is met with instructional feedback which builds an academic mindset that effort leads to achievement (i.e., grit)
3. **Conceptual understanding:** rather than math as a barrier to adventure games, ST Math games are built around the mechanics of math to promote powerful learning.
4. **Aligned:** the games can be aligned to the core curriculum creating a coherent instructional program (unlike most blended learning initiative) and allowing application of newly developed skills.
5. **ELL:** the games include no words so they are perfect for students new to English as well as advanced students.
6. **Support:** schools don't buy online games, they buy a proven program with strong implementation, professional development, and ongoing support.

# No tricks, and little memorizing!



Developing conceptual understanding



Comprehension of procedures/algorithms



Procedural skill and fluency

- <http://nixthetricks.com/>

# Conceptual Understanding:

- Teach more than “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives
- Students are able to see math as more than a set of mnemonics or discrete procedures
- Conceptual understanding supports the other aspects of rigor (fluency and application)
- Assessment problems - must allow students to access concepts from a number of perspectives to show deep understanding and allow error analysis

# This aligns with CCSS Wisdom:

## Three Shifts in Mathematics

- 1. Focus:** Focus strongly where the standards focus.
- 2. Coherence:** *Think* across grades, and *link* to major topics.
- 3. Rigor:** In major topics, pursue *conceptual understanding*, procedural skill and *fluency*, and *application*.

# NCTM Wisdom:



February, 2104

## Growth Mindset!

CCSS Math Practice #1:  
Make sense of problems and  
persevere in solving them.

Mathematics Teaching Practices	
→	<b>Establish mathematics goals to focus learning.</b> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
→	<b>Implement tasks that promote reasoning and problem solving.</b> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
→	<b>Use and connect mathematical representations.</b> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
→	<b>Facilitate meaningful mathematical discourse.</b> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
→	<b>Pose purposeful questions.</b> Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
→	<b>Build procedural fluency from conceptual understanding.</b> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
→	<b>Support productive struggle in learning mathematics.</b> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
→	<b>Elicit and use evidence of student thinking.</b> Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.





# Foster a Growth Mindset!

Sal Khan's Words of Wisdom

The Learning Myth:  
Why I'll Never Tell My Son He's Smart

<https://www.khanacademy.org/about/blog/post/95208400815/the-learning-myth-why-ill-never-tell-my-son-hes>

# Recommendation 4:

- Interventions should include instruction on **solving word problems** that is **based on common underlying structures**
  - Visual representations can be effective for teaching students how to categorize problems based on their structure and determine a solution method appropriate for the underlying structure

There are 21 hamsters and 32 kittens at the pet store. How many more kittens are at the pet store than hamsters?

32

21

?



# Math Story Problem Types:

[http://www.teachertipster.com/CGI\\_problem\\_types.pdf](http://www.teachertipster.com/CGI_problem_types.pdf)

JOINING PROBLEMS		
<b>Join (Result Unknown)</b> $6 + 3 = \underline{\quad}$	<b>Join (Change Unknown)</b> $4 + \underline{\quad} = 7$	<b>Join (Start Unknown)</b> $\underline{\quad} + 4 = 6$
Mr. Smith had 6 cookies. Suzy gave him 3 more cookies. How many cookies does Mr. Smith have now?	Mr. Smith had 4 cookies. Suzy gave him some more. Then, Mr. Smith had 7 cookies. How many cookies did Suzy give Mr. Smith?	Mr. Smith had some cookies. Suzy gave him 4 more cookies. Then, he had 6 cookies. How many cookies did Mr. Smith start with?

SEPARATING PROBLEMS		
<b>Separate (Result Unknown)</b> $7 - 4 = \underline{\quad}$	<b>Separate (Change Unknown)</b> $5 - \underline{\quad} = 1$	<b>Separate (Start Unknown)</b> $\underline{\quad} - 4 = 4$
Mr. Smith had 7 cookies. He gave 4 of them to Suzy. How many cookies did Mr. Smith have left?	Mr. Smith had 5 cookies. He gave some to Suzy. Then, he had 1 cookie left. How many cookies did Mr. Smith give to Suzy?	Mr. Smith had some cookies. He gave 4 to Suzy. Then, he had 4 cookies left. How many cookies did Mr. Smith have to start with?

PART - PART - WHOLE PROBLEMS	
<b>Part - Part - Whole (Whole Unknown)</b> $6 + 3 = \underline{\quad}$	<b>Part - Part - Whole (Part Unknown)</b> $7 - 4 = \underline{\quad}$ or $4 + \underline{\quad} = 7$
Mr. Smith had 6 white cookies and 3 pink cookies. How many cookies did Mr. Smith have altogether?	Mr. Smith had 7 cookies. 4 were pink and the rest were white. How many white cookies did Mr. Smith have?

COMPARING PROBLEMS		
<b>Compare (Difference Unknown)</b> $5 - 3 = \underline{\quad}$ or $3 + \underline{\quad} = 5$	<b>Compare (Quantity Unknown)</b> $3 + 2 = \underline{\quad}$	<b>Compare (Referent Unknown)</b> $8 - 5 = \underline{\quad}$
Mr. Smith had 5 cookies. Suzy had 3 cookies. How many more cookies did Mr. Smith have than Suzy?	Mr. Smith had 3 cookies. Suzy had 2 more cookies than Mr. Smith. How many cookies did Suzy have?	Mr. Smith had 8 cookies. He had 5 more than Suzy. How many cookies did Suzy have?




























MULTIPLYING AND DIVIDING PROBLEMS		
<b>Multiplication</b> $3 \times 3 = \underline{\quad}$	<b>Measurement Division</b> $9 \div 3 = \underline{\quad}$	<b>Partitive Division</b> $12 \div 3 = \underline{\quad}$
Mr. Smith had 3 piles of cookies. There were 3 cookies in each pile. How many cookies did Mr. Smith have?	Mr. Smith had 9 cookies. He put 3 cookies in each box. How many boxes did he need?	Mr. Smith had 12 cookies. He wanted to give them to 3 friends. How many cookies did each friend get?

\*WORD PROBLEM CHART BASED ON COGNITIVELY GUIDED INSTRUCTION PROBLEM TYPES

## Improving Mathematical Problem Solving in Grades 4 Through 8



[http://ies.ed.gov/ncee/wwc/pdf/practice\\_guides/mps\\_pg\\_052212.pdf](http://ies.ed.gov/ncee/wwc/pdf/practice_guides/mps_pg_052212.pdf)

Recommendation	Level of Evidence
<p>1. Prepare problems and use them in whole-class instruction.</p> <div>  <div>  Download Recommendation 1 (4.6 MB) </div> </div> <div>  <div>  Play Presentation (5:17 minutes) </div> <div>  Download Transcript (587 KB) </div> </div>	Minimal
<p>2. Assist students in monitoring and reflecting on the problem-solving process.</p> <div>  <div>  Download Recommendation 2 (4.6 MB) </div> </div> <div>  <div>  Play Presentation (4:58 minutes) </div> <div>  Download Transcript (540 KB) </div> </div>	<p>Strong</p> 
<p>3. Teach students how to use visual representations.</p> <div>  <div>  Download Recommendation 3 (4.6 MB) </div> </div> <div>  <div>  Play Presentation (4:51 minutes) </div> <div>  Download Transcript (557 KB) </div> </div>	<p>Strong</p> 
<p>4. Expose students to multiple problem-solving strategies.</p> <div>  <div>  Download Recommendation 4 (4.6 MB) </div> </div> <div>  <div>  Play Presentation (2:18 minutes) </div> <div>  Download Transcript (597 KB) </div> </div>	Moderate
<p>5. Help students recognize and articulate mathematical concepts and notation.</p> <div>  <div>  Download Recommendation 5 (4.6 MB) </div> </div> <div>  <div>  Play Presentation (2:23 minutes) </div> <div>  Download Transcript (921 KB) </div> </div>	Moderate


**Assist students in monitoring and reflecting on the problem-solving process.**

**Teach students how to use visual representations.**



# Thinking Blocks: Visual Models


**Thinking Blocks**  
Model and Solve Word Problems



The image shows a green oval containing eight rectangular blocks. The top row has four blocks: the first two show a green bar with a small orange segment added to the right; the next two show a blue bar with a small orange segment removed from the right. The bottom row has two blocks: the first shows a green bar with a small orange segment added to the right; the second shows a blue bar with a small orange segment removed from the right. To the right of the oval is a large green question mark. Below the oval is a plus sign and a minus sign.

**Addition and Subtraction Practice**  
Addition and Subtraction Word Problems

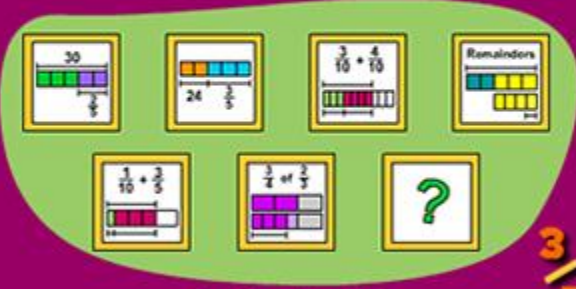
**Thinking Blocks**  
Model and Solve Word Problems



The image shows a green oval containing eight rectangular blocks. The top row has four blocks: the first two show a green bar with a small orange segment added to the right; the next two show a blue bar with a small orange segment removed from the right. The bottom row has two blocks: the first shows a green bar with a small orange segment added to the right; the second shows a blue bar with a small orange segment removed from the right. To the right of the oval is a large green question mark. Below the oval is a multiplication sign and a division sign.

**Multiplication and Division Practice**  
Multiplication and Division Word Problems


**Thinking Blocks**  
Model and Solve Word Problems



The image shows a green oval containing eight rectangular blocks. The top row has four blocks: the first two show a green bar with a small orange segment added to the right; the next two show a blue bar with a small orange segment removed from the right. The bottom row has two blocks: the first shows a green bar with a small orange segment added to the right; the second shows a blue bar with a small orange segment removed from the right. To the right of the oval is a large green question mark. Below the oval is a fraction  $\frac{3}{5}$ .

**Fraction Practice**  
Fraction Word Problems

**Thinking Blocks**  
Model and Solve Word Problems



The image shows a green oval containing eight rectangular blocks. The top row has four blocks: the first two show a green bar with a small orange segment added to the right; the next two show a blue bar with a small orange segment removed from the right. The bottom row has two blocks: the first shows a green bar with a small orange segment added to the right; the second shows a blue bar with a small orange segment removed from the right. To the right of the oval is a large green question mark. Below the oval is a ratio  $2:3$ .

**Ratio and Proportion Practice**  
Ratio and Proportion Word Problems

<http://www.mathplayground.com/thinkingblocks.htm>

## Recommendation 5:

- Intervention materials should include opportunities for students to work with **visual representations** of mathematical ideas
- Interventionists should be proficient in the use of visual representations of mathematical ideas

# Recommendation 5:

- The ability to express mathematical ideas using visual representations and to convert visual representations into symbols is critical for success in mathematics
  - Visual representations such as number lines, number bonds, strip diagrams/bar models, concrete drawings, and other forms of pictorial representations help scaffold learning and pave the way for understanding the abstract version of the representation
  - Interventionists should explicitly link visual representations with the standard symbolic representations: C-R-A progression

# The CRA Progression

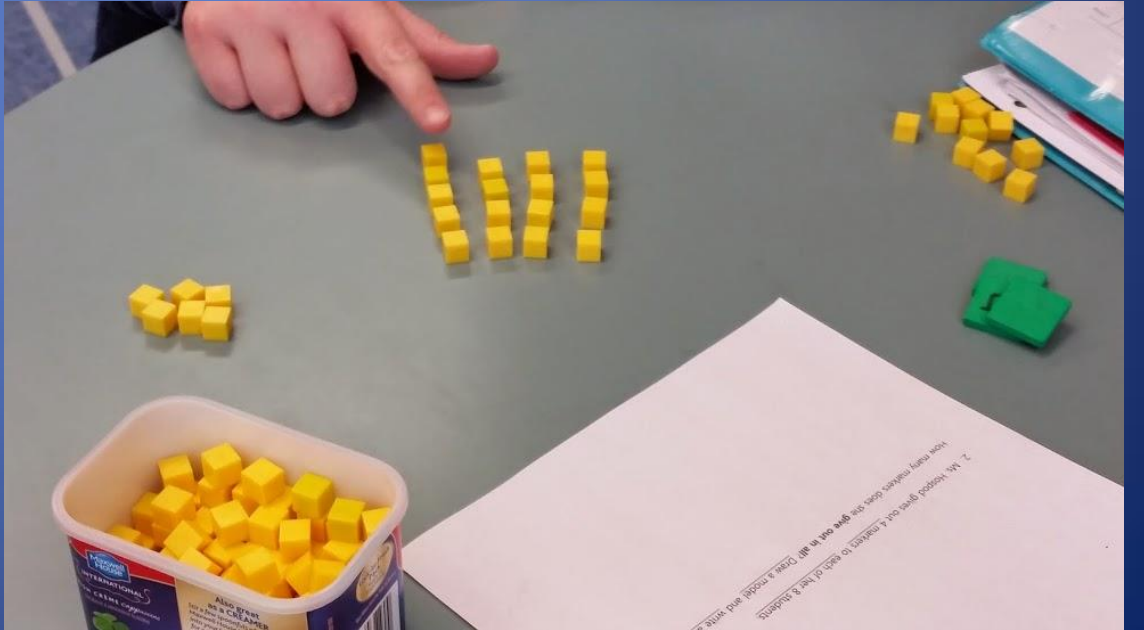
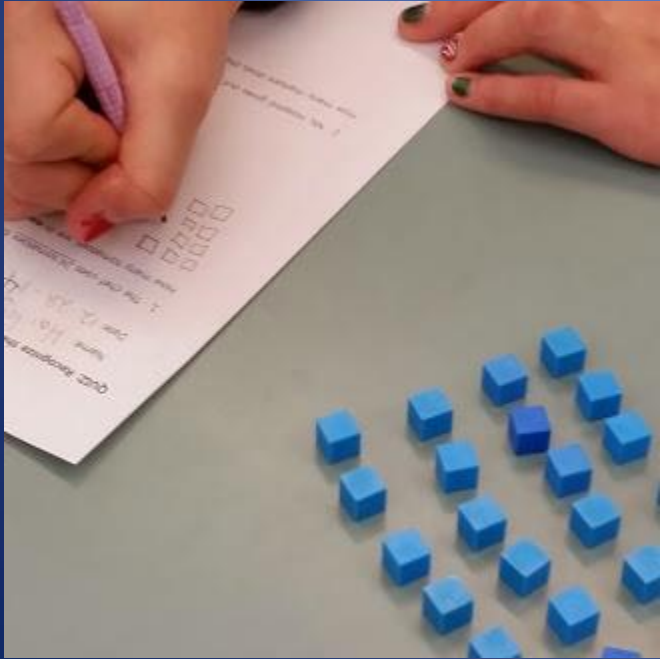
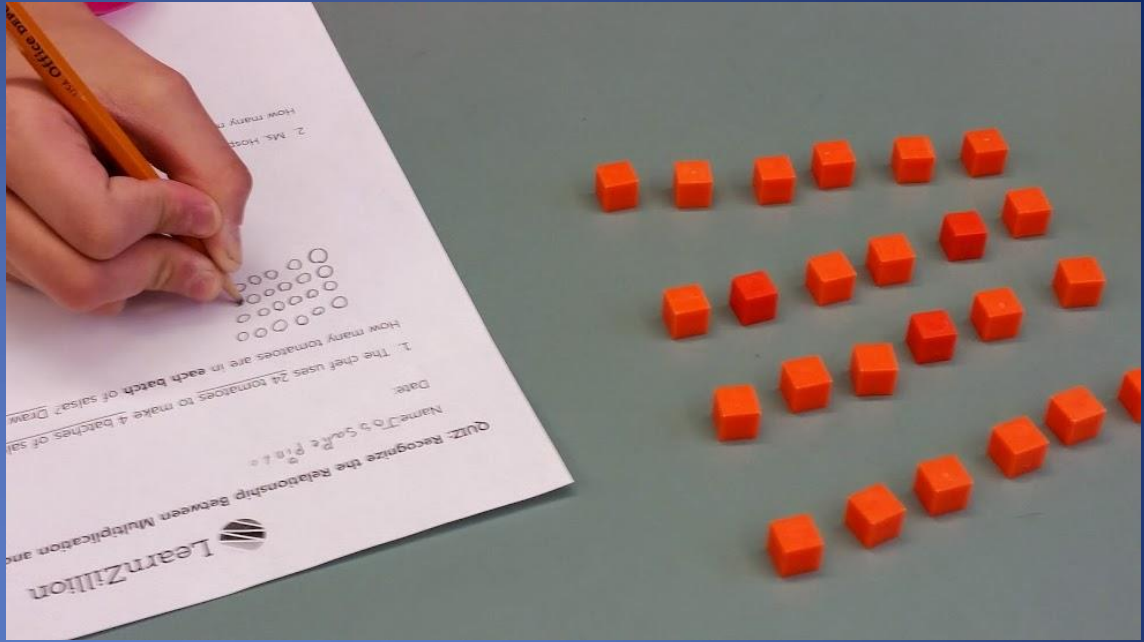
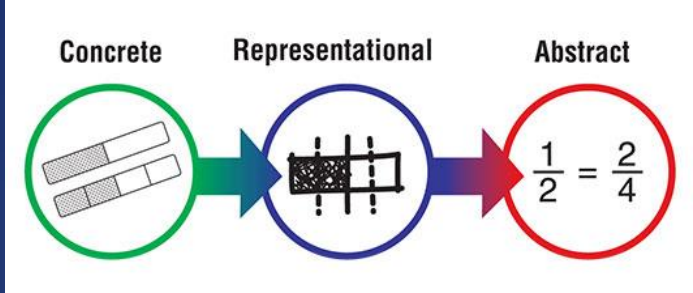
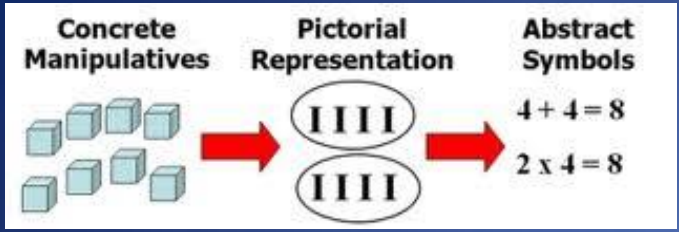
Concrete stage	Representational stage	Abstract stage
A mathematical concept is introduced with manipulatives; students explore the concept using the manipulatives in purposeful activity.	A mathematical concept is represented using pictures of some sort to stand for the concrete objects (the manipulatives) of the previous stage; students demonstrate how they can both visualize and communicate the concept at a pictorial level.	Mathematical symbols (numerals, operation signs, etc.) are used to express the concept in symbolic language; students demonstrate their understanding of the mathematical concept using the language of mathematics.

Research indicates that using manipulatives is especially useful for teaching low achievers, students with learning disabilities, and English language learners. (Marsh and Cooke, 1996; Ruzic and O'Connell, 2001)

Interventionists should allow students to continue to use manipulatives to demonstrate their understanding in the representational and abstract stages, if needed.

[https://www.hand2mind.com/pdf/learning\\_place/research\\_math\\_manips.pdf](https://www.hand2mind.com/pdf/learning_place/research_math_manips.pdf)







# Understanding the meaning of the divisor: Groups of 8 or 8 groups?

The whiteboard features two division problems written in red marker:

$$24 \div 6 = 4 \text{ in each group}$$
$$24 \div 6 = 4 \text{ groups in each}$$

Below these, the phrase "groups of 8" is written in large purple cursive script.

On the left side of the board, there is a vertical column of seven checkmarks (✓) next to the label "On Time".

On the right side, there is a vertical label "Total DAILY PTS - 10".

In the background, to the right, a poster is partially visible with the heading "When you are solving geometry problem..." and contains various formulas and instructions.

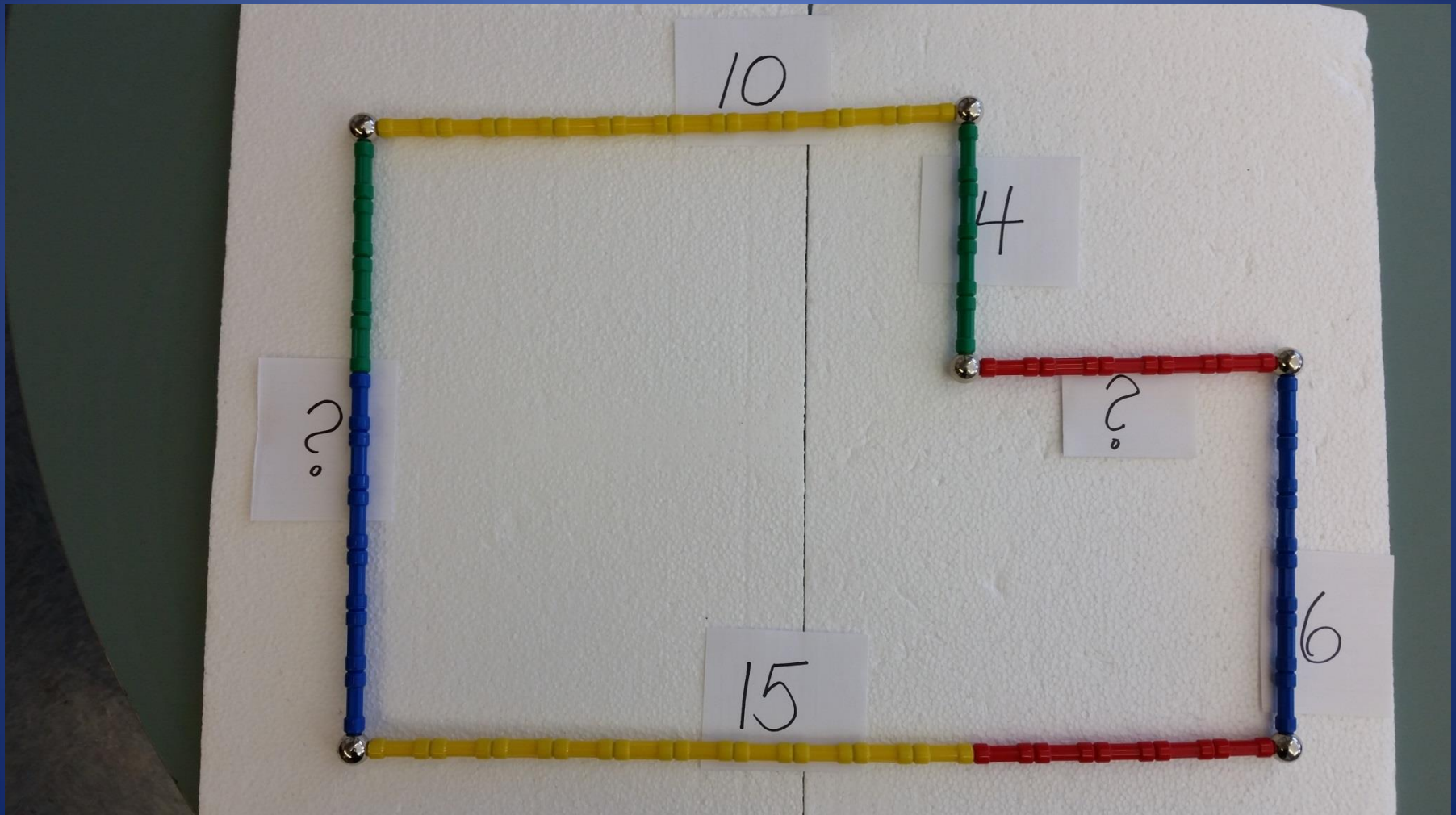




Groups of 8? Or 8 groups?




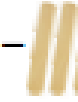
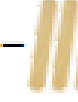













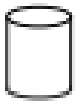
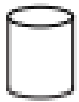




# Color Coding is Powerful!



**Example 8. A set of matched concrete, visual, and abstract representations to teach solving single-variable equations**

$$3 + X = 7$$

Solving the Equation with Concrete Manipulatives (Cups and Sticks)	Solving the Equation with Visual Representations of Cups and Sticks	Solving the Equation with Abstract Symbols
<p>A  +  = </p> <p>B  - </p> <p>C  = </p> <p>D  = </p> <p>E  = </p>	<p> +  = </p> <p> - </p> <p> = </p> <p> = </p> <p> = </p>	$3 + 1X = 7$ $\begin{array}{r} -3 \qquad -3 \\ \hline \end{array}$ $\frac{1X}{1} = \frac{4}{1}$ $X = 4$

**Concrete Steps**

- 3 sticks plus one group of X equals 7 sticks
- Subtract 3 sticks from each side of the equation
- The equation now reads as one group of X equals 4 sticks
- Divide each side of the equation by one group
- One group of X is equal to four sticks (i.e.,  $1X/\text{group} = 4 \text{ sticks/group}$ ;  $1X = 4 \text{ sticks}$ )



# Scaffold Support:

- Scaffolding supports students to reach new heights...



- But eventually the scaffolding must come down.

# *When it's Over*

by Ron F. Ferguson, Ph.D.

The lesson ain't over  
'til the skinny kid smiles  
and signals that he understands.

April, 2000

Ronald F. Ferguson, Faculty Co-Chair and Director, Achievement Gap Initiative at  
Harvard University and Founder, the Tripod Project for School Improvement

## Recommendation 6:

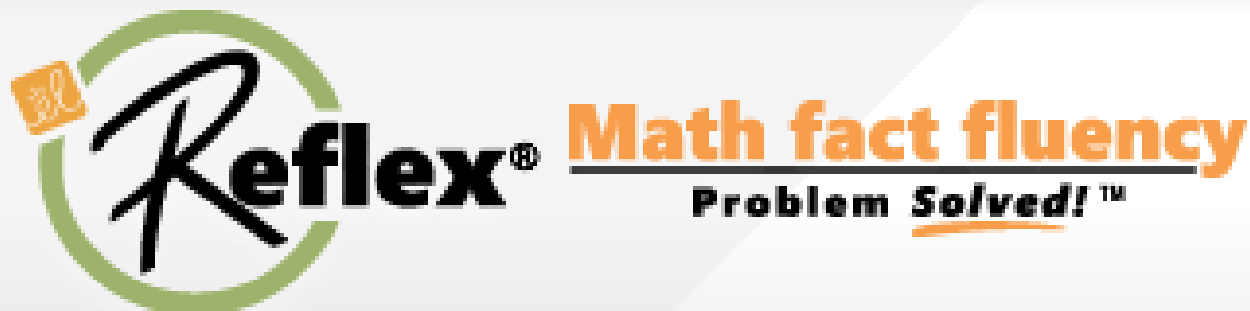
- Interventions at all grade levels should devote about 10 minutes in each session to building **fluent retrieval of basic arithmetic facts**.
  - Quick retrieval of basic math facts is critical for success in mathematics
  - Weak ability to fluently retrieve math facts impedes later understanding of rational number concepts



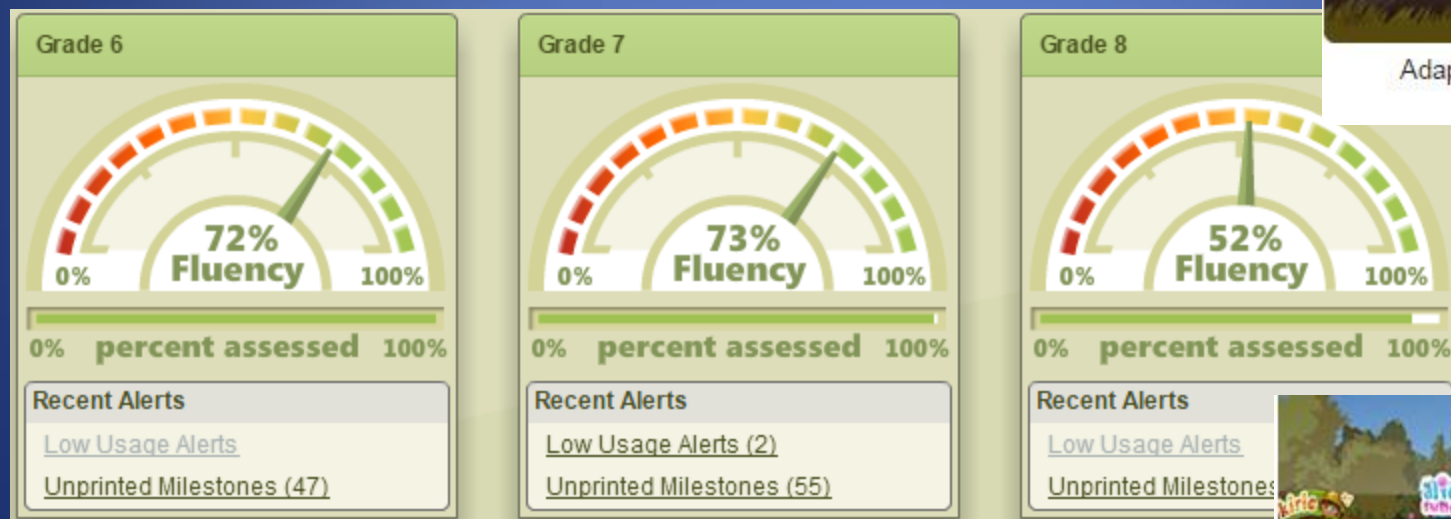
# Recommendation 6:

- The goal is quick retrieval of facts using the digits 0 to 9 without any access to pencil and paper or manipulatives
  - Fact families are an efficient way to learn
- In grades 2-8, also include instruction on how to use the **commutative**, **associative**, and **distributive** properties to derive more complex facts in their heads
  - **Number Talks** are great to develop this skill!

# Research-based Fact Fluency:



Adaptive and individualized instruction



<http://www.reflexmath.com/home>



Engaging games that keep students coming back for more

# Another Fact Fluency Program:

**FAST<sup>™</sup> Math**

**NEXT GENERATION**

District: Chittenden South Supervis    Server: CSSUFasttMath20

**FAST<sup>™</sup> Math**  
NEXT GENERATION

**STRETCH**

<http://teacher.scholastic.com/math-fact-fluency/fastt-math-next-generation/>

# This aligns with CCSS Wisdom:

## Three Shifts in Mathematics

- 1. Focus:** Focus strongly where the standards focus.
- 2. Coherence:** *Think* across grades, and *link* to major topics.
- 3. Rigor:** In major topics, pursue *conceptual understanding*, procedural skill and *fluency*, and *application*.

# Common Core State Standards

Table 1. Progress to Algebra in Grades K–8

K	1	2	3	4	5	6	7	8
Know number names and the count sequence	Represent and solve problems involving addition and subtraction		Represent & solve problems involving multiplication and division	Use the four operations with whole numbers to solve problems	Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions		
Count to tell the number of objects	Understand and apply properties of operations and the relationship between addition and subtraction	Represent and solve problems involving addition and subtraction	Understand properties of multiplication and the relationship between multiplication and division	Generalize place value understanding for multi-digit whole numbers	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Compare numbers		Add and subtract within 20	Multiply & divide within 100	Use place value understanding and properties of operations to perform multi-digit arithmetic	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems	Analyze proportional relationship and use them to solve real-world and mathematical problems	Understand the connections between proportional relationships, lines, and linear equations**
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	Add and subtract within 20	Understand place value	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Use properties of operations to generate equivalent expressions	Analyze and solve linear equations and pairs of simultaneous linear equations
Work with numbers 11–19 to gain foundations for place value	Work with addition and subtraction equations	Use place value understanding and properties of operations to add and subtract	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Define, evaluate, and compare functions
	Extend the counting sequence	Measure and estimate lengths in standard units	Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects			Represent and analyze quantitative relationships between dependent and independent variables		Use functions to model relationships between quantities
	Understand place value	Relate addition and subtraction to length	Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*			
	Use place value understanding and properties of operations to add and subtract							
	Measure lengths indirectly and by iterating length units							

# CCSS Math Practices for ALL Students:

## Standards for K-12 Mathematical Practice

1. Make sense of problems and persevere in solving them

6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

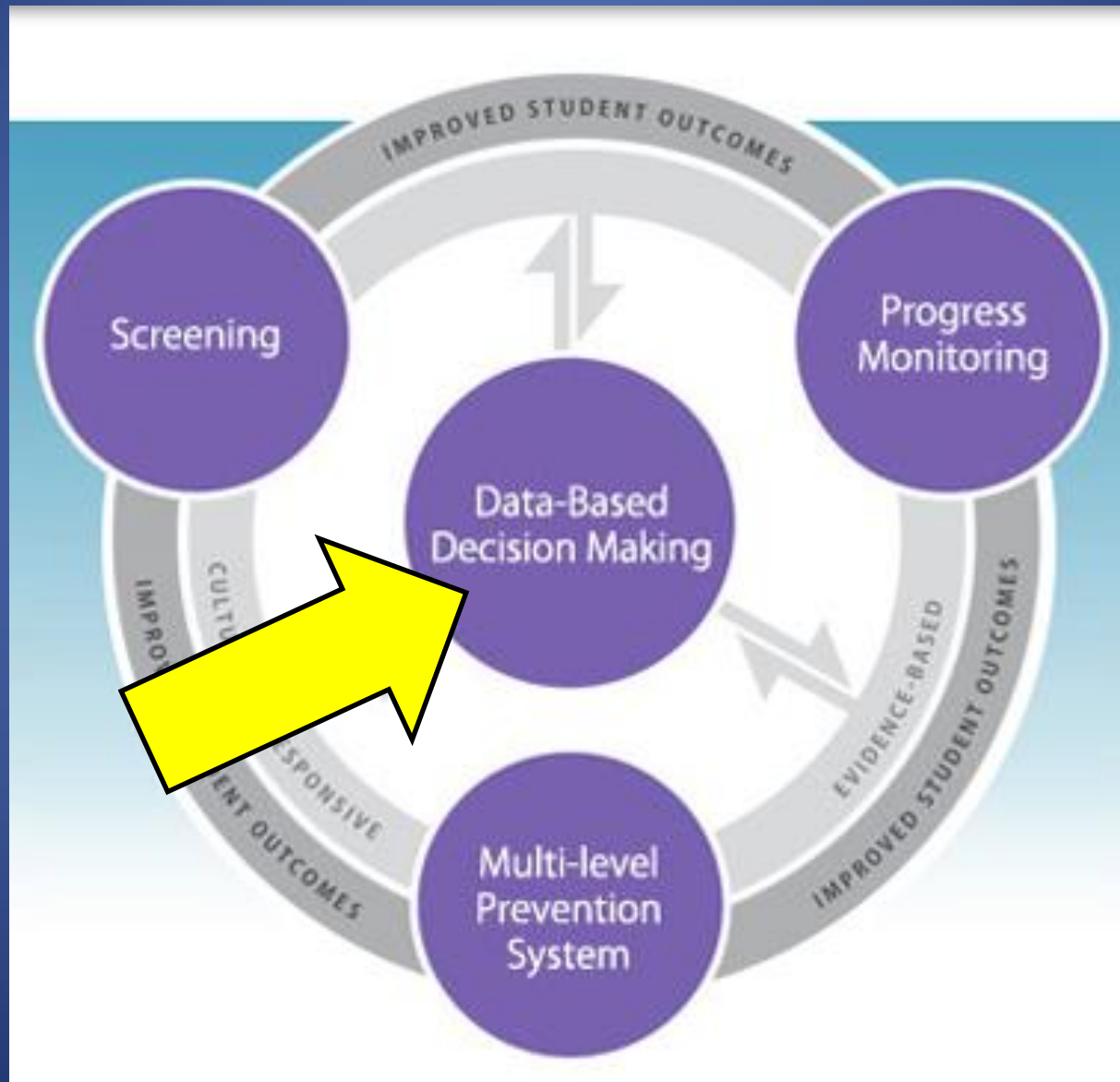
Reasoning and explaining

Modeling and using tools

Seeing structure and generalizing



# Essential Components of SRBI/RTI



# Data-Based Decision-Making

**Share Data**: with parents, with students – to inform, to motivate, goal setting



**Gather Data**: Fact fluency; Curriculum-Based Measures (CBM's) – specific skills; Conceptual understanding

**Examine Data**: analyze student work; specific skill focus; error analysis to inform instruction





# Professional Resources:

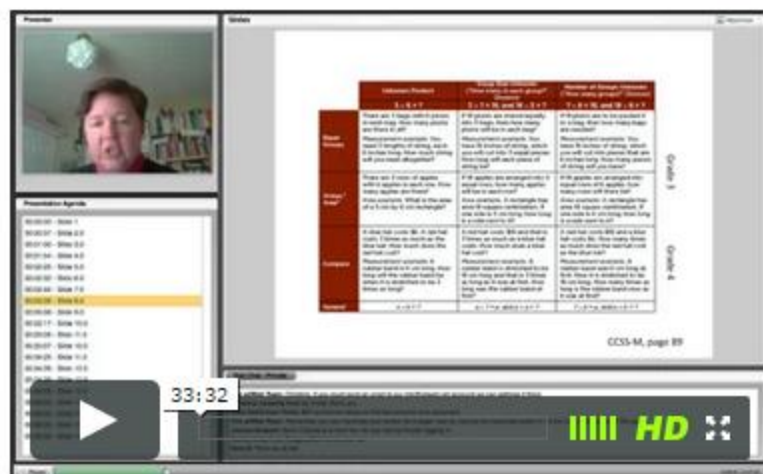


## Building Understanding in Mathematics

Community URL: [www.edweb.net/math](http://www.edweb.net/math)

Sponsored by ETA hand2mind

Preview a free webinar from this community!



**Building Understanding in Mathematics** is a free professional learning community (PLC) that provides a platform, advice and support in helping educators learn methods that help students build understanding in mathematics. In this forum you are encouraged to share resources, engage in discussions and collaborate with peers across the nation.

ACHIEVETHECORE.ORG

## JUMP TO A SECTION

Tasks

Assessments

Lessons

Tools for Planning

Understanding the Math

Professional Development

# Resources: What Works Clearinghouse

WHAT WORKS  
CLEARINGHOUSE

Home | Topics in Education | Publications & Reviews | Find What Works! | Inside the WWC | News & Events | About Us



### What Works in Math

Help your students achieve success in math with evidence-based resources from the What Works Clearinghouse™

### Evidence for What Works in Education

We review the research on the different programs, products, practices, and policies in education.

Then, by focusing on the results from *high-quality research*, we try to answer the question “What works in education?”

Our goal is to provide educators with the information they need to make **evidence-based decisions**.



### Publications & Reviews

**10,531** studies reviewed!

Get started with WWC products:

- **Practice guides** help educators address classroom challenges.
- **Intervention reports** guide evidence-based decisions.
- **Single study reviews** examine research quality.
- **Quick reviews** give the WWC's assessment of recent education research.
- The **studies database** contains all WWC-reviewed studies.



### Find What Works!

Intervention	Topic	Improvement Index	Effectiveness Rating	Extent of Evidence
Fast Forward® Language	English Language Learners	25	0	+
International Conversations and Literature Logs	English Language Learners	25	0	+

Based on the research evidence, find what works to...

- improve literacy skills in 3rd graders,
- increase math achievement in preschoolers,
- reduce dropout rates,
- help students with special needs,

... or address your school's challenge!



### WWC Fact Check: Test Your Knowledge...

“No discernible effects” means an intervention doesn’t work.

☐ True ☐ False



### What's New?

Subscribe to NewsFlash  and stay updated!

Watch our Webinar Designed for Faculty of Principal and Teacher Preparation Programs

<http://ies.ed.gov/ncee/wwc/default.aspx>



# Resources: RTI Action Network



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**What is RTI?**

**Research Support for RTI**

**LD Identification**

**Diversity and Disproportionality**

**Behavior Supports**

**RTI in Pre-Kindergarten**

**RTI in Secondary Schools**

Get Started ▶

Include Essential Components ▶

Connect With Others ▶

Professional Learning ▶

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Sign up to receive the RTI Action Network e-newsletter.

Email Address

**SIGN UP**

## What is RTI?



Response to Intervention (RTI) is a multi-tiered approach to help struggling learners. Students' progress is closely monitored at each stage of intervention to determine the need for further research-based instruction and/or intervention in general education, in special education, or both. [Read "What is RTI?" »](#)

## Why Adopt an RTI Model?

In the opening article for this section, David Prasse of Loyola University Chicago provides a historical context for RTI and a compelling argument for adopting an RTI model implemented with integrity in every school throughout the nation. Additional articles address RTI implementation in secondary schools and in content areas such as mathematics.

[Read "Why Adopt an RTI Model?" »](#)

## Approaches to RTI

Response to intervention (RTI) has a grassroots history with beginnings in multiple research areas. Over time, general categories of RTI implementations have emerged. They are briefly described in this article with guidance on selecting the right approach for schools and districts.

### WHAT'S NEW

[RTI-Based Specific Learning Disability \(SLD\) Identification Toolkit](#)

[Webinar: RTI-Based SLD Identification Toolkit](#)

[Considerations for English Language Learners](#)

[Case Study: Beginning with the Whole Mind - Florida's Systems Approach to Response to Intervention](#)

### FEATURED TERM

[Universal Design for Learning \(UDL\)](#)

<http://www.rtinetwork.org/learn/what>

# Resources: National Center on Intensive Intervention

National Center on  
**INTENSIVE INTERVENTION**

at American Institutes for Research ■

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## Tools Charts

**Academic Progress Monitoring Tools Chart**

**Academic Intervention Programs Tools Chart**

**Behavior Progress Monitoring Tools Chart**

**Behavioral Intervention Tools Chart**

**Behavioral Intervention Programs** – The 2014 call for tools is now closed.

**Behavioral Progress Monitoring Tools** – The 2014 call for tools is now closed.

**Academic Progress Monitoring Tools** – The 2014 call for tools is now closed.

**Academic Intervention Programs** – The 2014 call for programs is now closed.

## Review Information

- **Call Materials**
- **Review Process**
- **Technical Review Committee Members**

<http://www.intensiveintervention.org/resources/tools-charts>

# Resources: National Center on Intensive Intervention - Tools Chart

[Home](#) > [Tools Charts](#) >

## Academic Progress Monitoring GOM

This tools chart presents information about academic progress monitoring tools. The three tabs, *Psychometric Standards*, *Progress Monitoring Standards*, and *Data-based Individualization Standards* include ratings from our TRC members on the technical rigor of the tool. **Additional information** is provided below the chart.

View the [Progress Monitoring Mastery Measures](#) »

Grade Level

- Any - ▼

Subject

- Any - ▼

Apply

Psychometric Standards

Progress Monitoring Standards

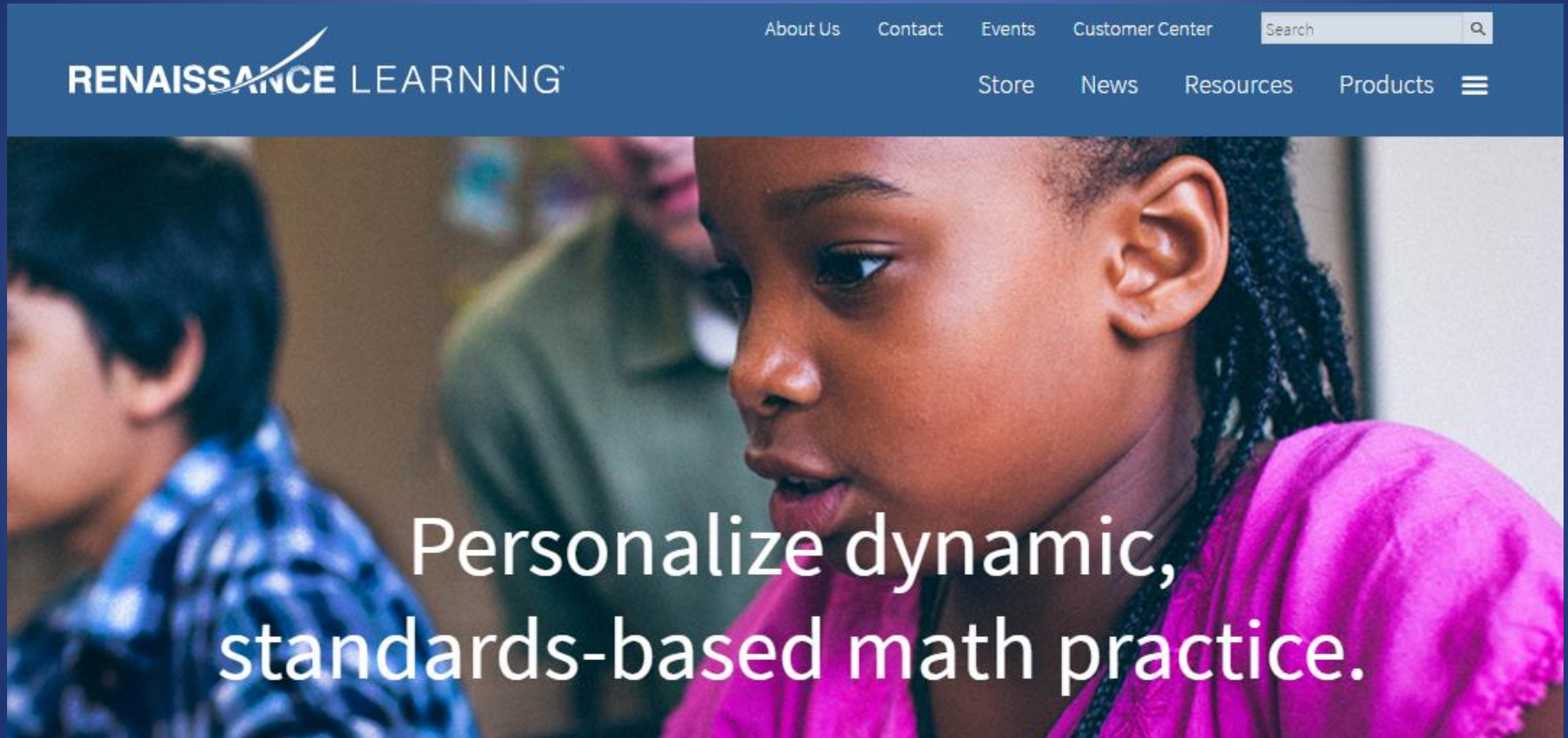
Data-based Individualization Standards

Title ▲	Area	Reliability of the Performance Level Score ⓘ	Reliability of the Slope ⓘ	Validity of the Performance Level Score ⓘ	Predictive Validity of the Slope of Improvement ⓘ	Disaggregated Reliability and Validity Data ⓘ
AIMSweb	M-CBM	●	●	●	●	●
AIMSweb	Math Computation	●	●	○	▬	▬
AIMSweb	Math Concepts and Applications	●	●	●	○	▬
AIMSweb	Oral Reading Fluency (R-CBM)	●	●	●	●	●
AIMSweb	Test of Early Literacy - Letter Naming Fluency	●	●	●	●	●
AIMSweb	Test of Early Literacy - Letter Sound Fluency	●	●	●	●	●

# RTI Implementer Series Self-paced Learning Modules

	<u><a href="#">Introduction</a></u>	<u><a href="#">Screening</a></u>	<u><a href="#">Progress Monitoring</a></u>	<u><a href="#">Multi-level Prevention System</a></u>
<u><a href="#">Defining the Essential Components</a></u>	<u><a href="#">What Is RTI? (34:25)</a></u>	<u><a href="#">What Is Screening? (18:09)</a></u>	<u><a href="#">What Is Progress Monitoring? (37:37)</a></u>	<u><a href="#">What Is a Multi-level Prevention System? (33:13)</a></u>
<u><a href="#">Assessment and Data-based Decision Making</a></u>	<u><a href="#">Understanding Types of Assessment within an RTI Framework (24:37)</a></u>	<u><a href="#">Using Screening Data for Decision Making (58:32)</a></u>	<u><a href="#">Using Progress Monitoring Data for Decision Making (53:10)</a></u> <sup>2</sup>	<u><a href="#">IDEA and the Multi-level Prevention System (10:57)</a></u>
<u><a href="#">Establishing Processes</a></u>	<u><a href="#">Implementing RTI (35:58)</a></u>	<u><a href="#">Establishing a Screening Process (12:40)</a></u>		<u><a href="#">Selecting Evidence-based Practices (53:46)</a></u>

# Renaissance Learning: Accelerated Math



<http://www.renaissance.com/products/accelerated-math>



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New

Find free  
video lessons



<https://learnzillion.com/resources/17132>

# Marilyn Burns' *Do the Math Now!*

The screenshot shows the top section of the 'Do the Math' website. At the top left is the 'SCHOLASTIC' logo. Below it is a red speech bubble containing the text 'Do The Math'. To the right of this is a large blue banner with the text 'DO THE MATH' in white, followed by four math symbols: a plus sign, a multiplication sign, a division sign, and a pie chart. Below these symbols is the text 'CREATED BY Marilyn Burns'. To the right of the banner is a blue box with the text 'New Do The Math Research Update' and a link 'read more >'. Below the banner is a sidebar with four blue buttons: 'DO THE MATH HOME', 'REDEFINING MATH INTERVENTION', 'DO THE MATH MODULES GRADES 1-6', and 'DO THE MATH NOW! GRADES 6 & UP'. Below the buttons is the text 'PROGRAM STRUCTURE'. To the right of the sidebar is a main content area with the title 'Do The Math Now!: Program Structure' and the subtitle 'Math Intervention Structured for Success'. Below the subtitle is a paragraph of text: 'Do The Math Now! provides step-by-step lessons organized into ten units, each with fifteen lessons that include teaching instructions, games, suggestions for differentiating instruction, and embedded assessment.' Below the paragraph is a table with three columns: 'Teaching for Understanding', 'Informing Instruction with Formative Assessment', and 'Measuring Student Understanding'.

**SCHOLASTIC**

**Do The Math**

**DO THE MATH**    

CREATED BY Marilyn Burns

New  
Do The Math  
Research  
Update  
[read more >](#)

DO THE MATH HOME

REDEFINING  
MATH INTERVENTION

DO THE MATH MODULES  
GRADES 1-6

DO THE MATH NOW!  
GRADES 6 & UP

PROGRAM STRUCTURE

## *Do The Math Now!:* Program Structure

### Math Intervention Structured for Success

*Do The Math Now!* provides step-by-step lessons organized into ten units, each with fifteen lessons that include teaching instructions, games, suggestions for differentiating instruction, and embedded assessment.

Teaching for Understanding	Informing Instruction with Formative Assessment	Measuring Student Understanding
----------------------------	--	---------------------------------

[http://teacher.scholastic.com/products/dothemath/dtmn\\_structure.htm](http://teacher.scholastic.com/products/dothemath/dtmn_structure.htm)

# Engage NY.org

engage<sup>ny</sup>

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engage<sup>ny</sup> > Common Core

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56 Results

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## Subject(s)

Math



Number & Operations --  
Fractions



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## Filters

Subjects ▾

☐ English Language Arts (2)

☒ Math

☐ Counting and Cardinality (1)

1 2 3 4 >



Grade 2 Mathematics  
Module 6, Topic B,  
Lesson 9

Curriculum Lesson



Grade 3 Mathematics  
Module 5, Topic F,  
Overview

Curriculum Topic



Grade 3 Mathematics  
Module 5, Topic E,  
Overview

Curriculum Topic



# *Persist*

by Ron F. Ferguson, Ph.D.

There is no greater frustration  
than to be stubbornly misunderstood  
by a child who is afraid that she can't learn.

And there is no greater elation  
than when the light of understanding  
burns away the fear and makes her smile return.

April, 2000



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Twitter : @CayFreeman

